



Cross (C. R.) & Goodwin (H. M.)

PROCEEDINGS
OF THE
AMERICAN ACADEMY
OF
ARTS AND SCIENCES.
VOL. XXVII.

PAPERS READ BEFORE THE ACADEMY.

I.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

XXXVII. — SOME CONSIDERATIONS REGARDING
HELMHOLTZ'S THEORY OF CONSONANCE.

By CHARLES R. CROSS AND HARRY M. GOODWIN.

Presented June 10, 1891.

THE present paper contains the results of a number of experiments relating to certain aspects of the theory of consonance and dissonance put forth by Helmholtz in the *Tonempfindungen*. In that work the author gives as the number of beats producing the maximum of harshness from thirty to forty per second, reaching this conclusion from a consideration of the amount of dissonance of various different chords in different octaves.

Mayer* studied the relation of the number of beats causing the greatest harshness to the absolute pitch, and showed that there is a marked rise in this number as the pitch becomes higher. The beats studied by Mayer, however, as he himself pointed out, are in several respects different from those which occur in the case of mistuned unisons. They were caused by rotating a disk, which

* American Journal of Science, Vol. CVIII. p. 241; Vol. CIX. p. 267.
VOL. XXVII. (N. S. XIX.) 1

LIBRARY
SURGEON GENERAL'S OFFICE

OCT. -10-1898

614

was provided with a circle of holes near the circumference, between a vibrating tuning-fork and a resonator, the number of beats per second being of course greater as the speed of the disk was greater. Hence the sound-wave reaching the ear was not at all a compound wave due to the superposition on one another of two simple sound-waves slightly differing in length, as in ordinary beats, but merely a series of simple waves with gaps, as it were; while the passage from maximum to minimum intensity in the sound reaching the ear was very different in its character from that actually occurring with ordinary beats. In referring to such beats as those studied by Mayer, we shall for the sake of brevity designate them as "interruptional beats," and the ordinary beats we shall call "interference beats."

We have thought it desirable to ascertain what results would be reached in different registers with interference beats, such as are present in actual dissonances. It was of course necessary to employ simple tones. Our method was a direct one, viz. to produce two such tones simultaneously, and to vary the pitch of one of these by known amounts, estimating by the ear when the sound became the harshest.

As a source of sound we employed the cylindrical resonators of Koenig, which were sounded by gently blowing across the opening a stream of air from the brass, slit-shaped "universal embouchure" made by the same maker. We found these tones to be very pure and loud, while their pitch could be varied with great readiness without altering the intensity of the sound emitted. The set of resonators employed ranged from Sol_1 to Mi_6 , the ranges of successive resonators overlapping slightly. The two embouchures were connected to a Y by rubber tubes, and placed on adjustable supports in suitable relation to the apertures of the two resonators to be sounded. The strength of the blast was regulated by a stopcock, and the bore of each rubber tube could be closed more or less by a clamp, so that the force of the jet of air at each resonator could be adjusted with all needed delicacy. Thus the intensity of the notes given out by the two resonators could be adjusted so that they were exactly equal in loudness. This was done by causing the tones to beat slowly, and so adjusting the apparatus that there was apparently an absolute silence when the sound fell to its minimum intensity. No difficulty was found from any whirring of the sheet of air as it struck the edges of the aperture, since by proper adjustment this could be made quite inappreciable. Having once made

the necessary adjustments, the pitch of either note could be varied by drawing out or pushing in the lower half of the resonator. The actual pitch of the note given by the resonator was in all cases determined by comparison with a tuning-fork of known rate. As the pitch of the resonator when blown is to a certain extent dependent on the position of the embouchure and the strength of the blast, it would not be safe to assume that the actual note given by the resonator was that marked upon it. Neither resonator when sounded sensibly affected the pitch of the note given by the other.

The determination of the point at which the harshness of the dissonance produced by the tones of the two resonators reaches a maximum is of course a very difficult problem to be solved exactly. It is, moreover, a question still to be investigated, whether precisely the same results would be reached by different observers possessed of equally good perception. The following figures given in Table I. were ascertained from a long series of observations by one of the writers (Mr. Goodwin). They are liable to an error of a single beat in either direction, possibly somewhat more in one or two cases. They furnish, however, the first series of direct measurements of this kind which we know.

TABLE I.

Ut_2	Ut_3	Sol_3	Ut_4	Sol_4	Ut_5	Mi_5
19	26	29	32	36	41	45

It will be noticed that the result for Ut_4 , 32 beats per second, agrees very closely with the number 33, which Helmholtz gives for maximum dissonance in the middle registers. He found that of the ordinary musical intervals, $b' c''$, as he designates it, was the most dissonant, and as this gave 33 beats per second he considered this number as giving the harshest effect for tones of corresponding pitch.

The notes furnished by the resonators are also well adapted for the direct study of the limit at which the beats cease to be audible when their number is increased. As the interval between the beating notes is increased, the beats heard become fainter and fainter until they disappear. It is all but impossible to fix upon the precise point of disappearance, since the change in loudness of the beats is so gradual. But, as will presently appear, our results clearly show that the maximum number of beats perceptible, like

the number giving the maximum dissonance, varies greatly with the pitch of the notes sounded.

The pitch of one of the resonators was varied so as to run upward and downward through the vanishing point of the beats. This procedure was extremely tiresome, and the values obtained were found to be considerably influenced by fatigue of the ear. The following table (Table II.) gives the results obtained when the ear was in good condition. The first column gives the rate of vibration of the lower note, the next four the rate of the upper note as found in corresponding series of observations; the seventh column gives the number of beats, and the last the interval between the two notes.

TABLE II.

Lower Note.	Upper Note.				Mean.	Beats.	Interval.
	1.	2.	3.	4.			
Ut_2 128	175	170	177	173	174	46	Fourth +.
Ut_3 256	328	321	326	329	326	70	Major third +.
Sol_3 384	467	458	469	460	464	80	Major third — 16 vibrations.
Ut_4 512	620	616	618	611	616	104	Minor third (nearly).
Sol_4 768	882	890	886	890	887	119	Tone + $\frac{1}{2}$ tone (nearly).
Ut_5 1024	1160	1167	1160	1163	1163	139	Tone + 11 vibrations.
Mi_5 1280	1420	1424	1421	1418	1420	140	Tone — $\frac{1}{8}$ tone (nearly).

These figures abundantly confirm for interference beats the results already reached by Mayer for interruptional beats, viz. that the ear is capable of appreciating a greater number of beats as the pitch of the beating notes rises. They also show very beautifully the increasing consonance of small intervals as the pitch rises, noted by Helmholtz.

The numerical results reached by us differ, as a whole, considerably from those given by Mayer for interruptional beats in either of his papers, though the divergence is less than that between the two series representing his own observations and those of Mrs. Seiler. In some cases, however, the agreement is very close. Thus, for Ut_2 we find the last trace of beats to occur when these are 46 in number. The number as observed by Mayer is 26, and as observed by Mrs. Seiler 45. For Ut_3 our figure is 70, and Mrs. Seiler's is 70 also.

For the other notes the divergence is wide, except for Ut_5 , for which our results agree quite well with those obtained by Mayer himself. These differences may be due to the different way in which the variations of intensity in the beats progress, to the presence of pitch variations in interference beats, to the fact that in our method the musical interval of the beating notes increases as the beats increase in number, to mere differences in the estimation of the observers, or to some other less evident causes.

We find that the ratios between the number of beats producing the greatest harshness at different pitches and the maximum number of beats discernible at those pitches are as follows (Table III.). The numbers given are obtained by dividing the former number by the latter.

TABLE III.

Ut_3	Ut_4	Sol_3	Ut_4	Sol_4	Ut_5	Mi_5
0.41	0.37	0.36	0.31	0.30	0.30	0.32

These figures, as a whole, agree very well with the ratio (four tenths) given by Mayer, although the absolute numbers whose ratios are taken are quite different from his. Our results also seem to indicate a clearly marked fall in this ratio as the pitch rises.

A few experiments were made using Professor Mayer's method, with some slight modifications, in order to observe the judgment of the same ear upon beats of different kinds. The tuning-fork employed was kept in vibration continuously by electricity, and the rotating disk was driven by an electro-motor, which secured great constancy of speed. We were seriously troubled by the sound made by the rotating disk, which produced a sort of siren effect as its openings passed before the aperture of the resonator. For Ut_3 the maximum number of intermittences perceived was 50 per second. For Ut_4 the number was about 106. These figures are liable to an error of perhaps two beats, or even a little more for the higher pitch. Mayer's figures are 47, 78, respectively, for his own observations on these notes, and 70, 130, for those of Mrs. Seiler. It is clear that there are very great differences in the sensitiveness of different ears for the perception of rapid beats.

In another series of experiments, interruptions were produced by a break-wheel placed in a telephonic circuit with a magneto-telephone at each end. The sound of a tuning-fork was transmitted through the line, and any desired number of interruptions could be produced by varying the rate of rotation of the wheel. Considerable annoy-

ance was experienced at times from sounds which apparently arose from microphonic action at the break-wheel. These can probably be avoided in future experiments by causing the wheel to divert the current from the receiver periodically, instead of interrupting it. The limits at which the beats disappeared, with the notes experimented upon, were as follows: Ut_4 48, Mi_4 56, Sol_4 69. These figures are the means of a number of observations, and are apparently true within less than one beat per second. Comparing this kind of interruptional beat with those given by Mayer's method, the maximum number perceptible seems to be smaller for the former than for the latter. This is probably due in part to the greater loudness of the sounds observed in the latter case, but there is another marked difference which must exercise an important influence upon the phenomenon under consideration. With the beats produced by the break-wheel the sound begins and ends almost instantaneously, so that sharply marked intervals of sound and silence of equal duration succeed each other. One would naturally expect the telephonic beats to be the more distinct of the two, which is not the case. The subject is one which requires further study as to the influence of the relative duration of sound and silence in the telephonic method, and of the relative size of the apertures in Mayer's method.

In the matters which we have thus far discussed in the present paper, our results are fully in accordance with the theory of consonance proposed by Helmholtz. In some other particulars, however, we are led to conclude that this theory is incomplete, at least.

Some years ago, in a paper read at the Philadelphia meeting of the American Association for the Advancement of Science, and of which an abstract was published in the Proceedings of that Society for 1884 (page 113), one of us called attention to the bearing of certain phenomena of binaural audition upon Helmholtz's theory as follows:—

“In connection with his study on the effects of beats in causing dissonance, Helmholtz considers the condition of the vibrating portions of the inner ear as to resonance and damping. In his remarks on the subject, he assumes that no notes are capable of beating with each other unless they both affect the same vibrating element of the inner ear. This view of the matter gives a purely mechanical action in the ear as the explanation of the physiological phenomenon of beats.

“In addition to the experiments of Koenig, in which beats were

obtained between notes of very great intervals, there are certain phenomena of binaural audition which appear to prove that this view is incorrect, and that rather than, or at least in addition to, this mechanical interference of vibrations in the ear, there is a more obscure operation within the sensorium itself. I refer to the fact that beats may occur when the exciting sounds operate upon different ears, and also apparently between the after-sensation and the succeeding sound of mistuned unisons. Whether this last phenomenon can be observed when the after-sensation results from an impression made upon the opposite ear to that which is impressed by the succeeding sound, I do not know; but in view of the fact that two simultaneous sounds acting upon different ears may beat, I do not see why this result should not be possible.

“Now if beats arise between the after-sensation and a following sound, Helmholtz’s view can be true only upon the supposition that the vibrating parts of the inner ear continue in motion as long as the after-sensation persists; and that there is no residual sensation capable of giving beats other than that which persists only as long as the actual vibration in the ear itself continues. Such a supposition seems quite improbable. Moreover, the phenomenon of beats in the case of sounds acting simultaneously upon different ears cannot be explained even upon this supposition. There must be some kind of *vibration*, using this term in the most general sense, or some kind of alternation of phase or state within the sensorium itself.

“There is no question as to the fact of the production of beats under the circumstances last mentioned. I have not only verified the fact by experiments conducted in the usual manner, but have also studied the beats produced when a tuning-fork is held close to one ear and the sound of a second fork, not quite in unison with it, is transmitted by telephone to the other, taking suitable precautions that the ear against which the receiving telephone was held was not affected by the sound of the neighboring fork. Beats were readily obtained, which grew weaker as the fork was moved away from the open ear and approached to the ear which was closed by the receiver. The same result occurred when, instead of mistuned unisons, harmonic forks were used, which gave beats with the fundamental.”

In the experiments just cited, the fact that the fork gave feeble beats as it was moved towards the ear to which the telephone was applied seemed clearly to indicate that the beats were not likely to be due to sounds transmitted through the head from one ear to the

other, or to the passage of the vibrations through the skull from the outer air. These experiments did not differ in other respects from those of previous observers. The beating of the sound persisting in one ear after excitation had ceased with a second sound falling upon the other ear was, in fact, observed by S. P. Thompson, in 1881.

The existence of such binaural beats as are under consideration was noted as early as 1874 by Mach, who assumed that the sound was conducted from one ear to the other through the bones of the head. S. P. Thompson has studied the subject very carefully;* and while he is evidently strongly inclined to believe that the phenomena are due to an interference of sensations not produced by the same ear, nevertheless he does not appear to consider this as a fact finally settled beyond all question, although to us his results seem to leave small room for doubt on the matter.

The important bearing of the existence of binaural beats upon some parts of Helmholtz's theory of audition does not seem to have been very fully recognized. This may be because of some question as to whether each sound did not after all really act upon both ears, even when apparently applied to one alone. From the nature of the case, it is very difficult to prove finally that there is no possibility of such conduction of sound from one ear to the other, but we have sought to study the phenomena of binaural beats under circumstances which render such conduction improbable in the highest degree. With this end in view, we have repeated most of the experiments of Thompson, and made such additional tests as suggested themselves to us.

The sounds of two tuning-forks placed in a distant room were conveyed to the ears telephonically over two separate circuits. These forks were struck by hammers actuated by electro-magnets placed in circuits separate from those containing the telephones, and governed by keys operated by the foot of the observer at the receiving end of the line. This method was found to be preferable to running the forks continuously by electro-magnetic means, as the currents used in the latter method were liable to act directly upon the transmitting telephone and cause disturbing noises. Very powerful magneto-telephones were used as transmitters and receivers. *Mi*₄ forks were used throughout the experiments, unless otherwise

* Philosophical Magazine, Vol. IV. p. 274; Vol. VI. p. 383; Vol. XII. p. 351; Vol. XIII. p. 406.

stated, on account of the remarkable intensity with which their tones were transmitted, this being approximately the natural tone of the diaphragm. The singular phenomena of the localization in the back of the head of the sensation caused by two notes in opposite phases when heard by separate ears, and the apparent wandering of the sensation from one ear to the other in the case of binaural beats referred to by Thompson, were always clearly recognized.

The method of experimentation precludes any fusion of the two sounds, except within the head; but to decide whether the beats arise from an interference of the sensations or from a mechanical interference in Corti's organ, we must investigate further. We tried to throw some light on this point by the following experiments. It is well known that the vibrations of a fork, even when of too small amplitude to be heard when held opposite the ear itself, can be made audible by pressing the stem of the fork against the bone of the skull, or, still better, against the teeth, the latter transmitting vibrations to the ears better than any other part of the head. A still more effectual means of making audible very small vibrations is to close the ear with a bit of beeswax and press the stem of the fork lightly against the wax. In this case the vibrations are transmitted to the membrana tympani by the small amount of air enclosed within the meatus, as is clear from the fact that the sound of the fork is heard on touching the wax long after it ceases to be audible on touching its stem to the pinna of the ear. Hence in this case there is no conduction to the middle or inner ear through the bones of the head. Whatever sound reaches the ear follows the ordinary path through the meatus to the membrana tympani. Now we found that the vibrations of a fork could be heard longer when touched to the wax in the ear than when held against the teeth. We therefore took two small tonometer forks making four beats per second, struck them very gently, and held their stems against the teeth; loud beats were heard in the ears just as they are ordinarily heard when powerful forks are sounded in the air. The forks were then held in this position until the beats had entirely ceased to be audible, when they were removed, and the stem of each was touched to the wax closing the two ears. Instantly the two notes were heard, faintly but distinctly, in the ears to which they were held, and accompanying them were faint beats seeming to wander in the head from ear to ear, as is always the case with binaural beats. In trying this experiment care was taken that no vibration should be imparted to the fork from a gentle blow given on touching its stem to the wax,

and it was found that the liability of such an occurrence could be greatly diminished by covering the end of the stem with a layer of wax.

To make sure that these beats were not the result of imagination, we took forks giving four, eight, twelve, and sixteen beats per second at random, struck them lightly as before, but in this case held only one of them against the teeth, so as not to know the number of beats which ought to be produced. When this fork had ceased to be heard, we touched both forks to the wax in the ears, as before, and in every case the correct number of beats was at once heard. The experiment was varied slightly as follows. One ear only was closed with wax; the other was immersed in a large basin of water. The experiment was then repeated as above, with the difference that one fork, instead of being touched to the ear, was touched to the marble basin, its vibrations being transmitted to the enclosed ear through the water. The same results were obtained as before. These experiments were all carried on at night, when there was not the slightest disturbing vibration in the air. Together with another experiment described below, they lead us to conclude that aerial vibrations acting upon the ear are not transmitted through the skull or bony parts of the head from one ear to the other. The experiments cited certainly seem to bear out this view, at least for tones of feeble intensity. Furthermore, the fact that differential tones have not been produced binaurally leads us to believe that it is true generally, even for very intense sounds, that binaural beats do not result from sound conduction within the head.

In his experiments Thompson was never able to obtain differential tones unless the sounds were allowed to mingle before falling upon the ears. Our experiments fully bear out this result, which we have verified in the following ways: (1) By listening to Ut_4 and Mi_4 forks through the telephones. These notes were transmitted with more than sufficient intensity to give rise to a differential when heard in the ordinary manner; but with a telephone placed at each ear, not the faintest trace of such a tone could be heard. We also reached equally negative results on listening to Mi_4 with a telephone, and holding an Ut_4 , Sol_4 , or Sol_3 fork close to the other ear, or on leading their vibrations to the ear through a long rubber tube. (2) By Thompson's method of carrying the tones to the ears by means of rubber tubes (in our experiments about thirty feet long), the forks being struck, one in another

room and the other outside of the window, to prevent their tones from mixing before reaching the ears. No differential tone could be heard. We also tried to use Koenig's forks for illustrating beat-notes, but their very rapid vibrations were so deadened by the tubing that they could only be heard very faintly. (3) By the same process as above, with the exception that the ends of the rubber tube were connected with cylindrical resonators which were blown as already described. This gave almost deafening tones in each ear, but not the slightest trace of a differential. But in this case, as also in (2), if the two tubes were connected to two branches of a Y tube, and a third rubber tube leading to the ear was connected to the third branch, the differential tone came out at once. (4) The most striking proof was the following. The ears being closed with wax, a brass rod about five feet long was held lightly against the wax in each. When the stems of forks struck by two assistants were pressed against the farther end of the rods, very loud tones were heard in the ears, unaccompanied by any differential tone. If, however, one of the rods was removed from the ear and pressed tightly against the head, or, better, against the teeth, a loud differential tone was heard at once in the ear against which the rod was placed. If both rods were held against the teeth or head, the differential tone was heard in both ears.

This apparent impossibility of producing differential tones when the primary tones are caused to actuate separate ears would seem conclusive of the point under discussion except for the fact that it might reasonably be argued that, if sound conduction through the skull occurred from one ear to the other, the vibrations thus conducted might act directly on the inner ear, and not indirectly through the membrana tympani and chain of bones, in which case no differential tones should arise, although beats could still take place in the manner assumed by Helmholtz. But our experiment with the brass rods, described above, shows that such tones are in fact readily produced when conduction to the ear takes place through the bones of the head. And if there were any material conduction of this kind, it certainly seems as if the very loud sounds used by us would have given evidence of it at least by the occasional production of differential tones when the sounds actuated separate ears.

The production of beats without the simultaneous action of the beating sounds upon the same ear seems, therefore, to be clearly shown from these facts: (1) Beats are produced by sounds falling

upon the ears separately even when the greatest precautions are taken to prevent both from affecting the same ear. (2) Under no circumstances does it seem to be possible to produce differential tones by binaural combination, even when the separate tones are extremely loud. (3) In all cases in which the beating sounds have apparently acted separately upon the auditory apparatus of each ear, the singular phenomenon of encephalic localization of the beats has appeared, while this phenomenon has never been found present either when the two sounds acted simultaneously upon the same ear through the auditory canal, or when one of them was manifestly transmitted to the ear through the bones of the skull.

We are therefore led to the conclusion that the mechanism in which the interference causing beats takes place, and through the operations of which we consequently recognize the character of consonant and dissonant intervals, lies, in part at least, beyond the auditory apparatus of the inner ear. It seems to us clear that notes may beat when they do not affect the same vibrating elements of the inner ear, and that we must look to some more profound changes within the sensorium itself for a complete explanation of the phenomena under consideration.

It may be possible that there are two distinct causes of beating; one, that suggested by Helmholtz, which wholly or principally obtains when the two notes act simultaneously upon the same ear, and the other due to some different and more obscure action which obtains in the case of binaural beats. In this case quite different laws as to consonance might hold. The exceeding harshness of dissonances in binaural audition lends some support to this view.

ROGERS LABORATORY OF PHYSICS,

May, 1891.